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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2017

FIRST YEAR [BATCH 2017-20]

MATHEMATICS FOR ECONOMICS [General]

Paper : I

Date : 23/12/2017

Time : 11 am – 2 pm

Full Marks : 75

[Use a separate Answer Book for <u>each Group</u>]

<u>Group – A</u>

Answer <u>any five</u> questions from <u>Question Nos. 1 to 8</u> : [5×7]

1. a) Let $A=\{1,3,5,10,12\}$, $B=\{2,4,6,8,10\}$ and $C=\{5,4,6,10,15,17\}$.
   Then find— 2½ × 2
   (i) $(A\cup B)^c \cap C^c$
   (ii) $(A-B)^c \cup (A-C^c)^c$

   b) If $A=\{1,2\}$, $B=\{a,b\}$ then find $A\times(B\cup A^c)$. 2

2. a) Show that the map $f:Q\to Q$ defined by $f(x)=3x+2$ is one-to-one, where Q is the set of rational numbers. Also find a formula for f^{-1}. 4

   b) Show that the mapping $f:I\to I$ defined by $f(x)=x^2,\ x\in I$, where I is the set of positive integers, is one-one into. 3

3. a) Use principle of mathematical induction to show that, $(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$. 4

   b) Prove that $\sqrt{5}$ is irrational. 3

4. a) Prove that arbitrary union of open subsets of $\mathbb{R}$ (set of all real numbers) is an open set. 4

   b) Give an example of a set which has exactly three limit points. 3

5. a) Define Cauchy Sequence and bounded sequence in $\mathbb{R}$. 3

   b) Give an example of a bounded sequence which is not a Cauchy sequence. 2

   c) If $\{a_n\}_n \to a$ then prove that $\{ca_n\}_n \to ca$, where c is a constant. 2

6. a) If $a_n=\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n(n+1)}$, then find $\lim\limits_{n\to\infty} a_n$. 4

   b) Find $\lim\limits_{n\to\infty}\left(\sqrt{n-1}-\sqrt{n}\right)$, if exists. 3

7. a) Define Cauchy criteria for convergence of an infinite series. 2

   b) Find the sum of the series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\cdots$ 5

8. Find whether the following series is convergent or divergent: $3\frac{1}{2} \times 2$

a) $x^2 + \dfrac{2^2}{3\cdot 4}x^4 + \dfrac{2^2\cdot 4^2}{3\cdot 4\cdot 5\cdot 6}x^6 + \dfrac{2^2\cdot 4^2\cdot 6^2}{3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8}x^8 + \ldots, \quad (x > 0),$

b) $1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} \cdots\cdots$

<u>Group – B</u>

***Answer <u>any five</u> questions from <u>Question Nos. 9 to 16</u>* :** [5×6]

9. a) Calculate the value of $\dfrac{\left(\cos\dfrac{\pi}{22} + i\sin\dfrac{\pi}{22}\right)^{11} \times \left(\cos\dfrac{\pi}{21} - i\sin\dfrac{\pi}{21}\right)^{7}}{\left(\cos\dfrac{\pi}{36} + i\sin\dfrac{\pi}{36}\right)^{12}}$. 2

b) If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then prove that

$$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$$

and $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$. 4

10. a) Show that the set $\mathbb{Z}$ of all integers does not form a group under the operation $x * y = x - y$ for every $x, y \in \mathbb{Z}$. 2

b) Show that the set of matrices of the form $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$, is a ring where a, b, c are real numbers. 4

11. a) Verify whether the set of all rational numbers forms a group under the operation $x * y = x + y + 1,\ x, y \in \mathbb{Q}$ (set of all rationals). 2

b) Verify whether the set of all 2X2 real orthogonal matrices form a ring over the set of all real numbers. 4

12. a) Give an example of a ring which is not a field. 3

b) If a, b be two elements of a field F and $b \neq 0$, then prove that a = 1, if $(a\cdot b)^2 = a\cdot b^2 + bab - b^2$. 3

13. a) Define singular and non-singular matrix. 2

b) State the necessary and sufficient condition for a matrix to be invertible. 1

c) Let, $A = \begin{bmatrix} 2 & 3 \\ 1 & 7 \end{bmatrix}$ then show that $A(adj\ A) = (adj\ A)\ A = \det A \cdot I$ where, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. 3

14. a) Show that $\begin{vmatrix} a_1\alpha_1 + b_1\beta_1 & a_2\alpha_1 + b_2\beta_1 & a_3\alpha_1 + b_3\beta_1 \\ a_1\alpha_2 + b_1\beta_2 & a_2\alpha_2 + b_2\beta_2 & a_3\alpha_2 + b_3\beta_2 \\ a_1\alpha_3 + b_1\beta_3 & a_2\alpha_3 + b_2\beta_3 & a_3\alpha_3 + b_3\beta_3 \end{vmatrix} = 0$. 3

b) Show that $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$ is a perfect square. 3

15. a) If $A=\begin{bmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{bmatrix}$, then find A^2 and show that $A^2=A^{-1}$. 3

b) If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, then show that $A^2-4A-5I=0$ is satisfied, where I is a 3x3 unit matrix. 3

16. a) Define equivalent matrices. 2

b) Solve the system of linear equation 4

$$x+z=1$$
$$y+3z=5$$
$$x+y+2z=8$$

Answer <u>any two</u> questions from <u>Question Nos. 17 to 19</u> : [2×5]

17. Find the non-singular matrices P and Q, such that PAQ is in normal form and hence find the rank of the matrix A, where $A=\begin{bmatrix}1 & 1 & 1\\ 1 & -1 & -1\\ 3 & 1 & 1\end{bmatrix}$. 3+2

18. a) Find $\Delta^2\left(e^x+\frac{1}{x}\right)$, $h=1$. 3

b) Find $\frac{1}{E+1}\left(x^2+2x+3\right)$, $h=1$. 2

19. Solve the difference equation, $u_{x+2}+u_{x+1}-12u_x=5^x$, $x\geq 1$. 5

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